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Application of He's Variational Iteration Method to Solve Convection Diffusion Problems

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Abstract: **In this paper, VIM is used to explain convection diffusion problems in physical situations where energy, particles and other forcible amount are transmitted inside a physical system owed to diffusion and convections. The numerical results obtained by VIM are discussed with the help of different figures. The plotted graph and numerical results show accuracy and efficiency of this method in solving nonlinear equations.**

Keywords: **Variational iteration method; Convection diffusion problems; Kolmogorov-Petrovsly - Piskunov (KPP) equation.**

1. INTRODUCTION

Nonlinear phenomena are always visible in the study of applied Mathematics, Physics, Chemistry and many related fields of engineering. Solving nonlinear system is a herculean task in mathematical analysis and applications. In daily life, we come across many real life models of Mathematics for numerical solution of nonlinear differential equations. The significance of obtaining their exact solution, if available, facilitates the authentication of numerical solvers as well as supports in stability analysis of the solution. Analytical solutions to nonlinear PDEs play a vital role in nonlinear physical science as they can offer more physical information as well as better insight into the physical aspects of the problem and hence lead to further applications [1]. Various methods have been used for the solution of nonlinear partial differential equations by many researchers in the last two or three decades. Some notable of them are inverse scattering method [2], Adomian decomposition method [3], Laplace decomposition method [4], Homotopy perturbation method [5], Differential transform method [6], Homotopy analysis method [7], exp-function method [8], Variational iteration method [9–11] etc.

The convection-diffusion problem is the governing equation of many important transport phenomena in building physics. The convection-diffusion equation in general form is

$$\frac{\partial u}{\partial t} = \nabla.(D.\nabla u) - \nabla.(\vec{v}u) + R,$$

Combined transport of a substance through a porous medium caused by diffusion and convection is a relatively frequent problem in building physics. Such problems are the centre of many recent investigations not only due to their importance in building physics analyses, but also due to some problems with numerical stability in their solution. The diffusion-convection equation elaborates heat - flow, particles and other active quantities in position. The diffusion is caused by concentration gradients, sector in a gas mixture. In case of the motion of bulk fluid, the flux of chemical species and convection are contributed.

Gupta et al. [12-13] presented homotopy perturbation transformation method (HPTM) to solve various convection-diffusion problems. Ghasemi and Kajani [14] applied homotopy perturbation method (HPM) to solve linear and nonlinear diffusion-convection problems. Liu and Zhao [15] applied variational iteration method (VIM) to solve one dimensional unsteady convection-diffusion equations. Momani [16] applied Adomian decomposition method (ADM) to solve nonlinear fractional convection-diffusion equation. Yuzbasi and Sahin [17] presented numerical scheme of Bessel collocation method for the approximate solution of the one-dimensional parabolic convection-diffusion model. Wakil et al. [18] used ADM to solve explicit numerical solution of three types of the diffusion–convection-reaction equations. Odibat, Jafari and Gejji [19, 20] used ADM to solve fractional diffusion-wave equation. Hashim [21] presented two-dimensional diffusion problem solved by ADM and a IV order explicit finite-difference method. Chen et al. [22] used wavelet method to solve a class of space-time fractional convection-diffusion equation with variable coefficients. The VIM was proposed by J. H. He [23-25] and then applied it to solve autonomous ordinary differential equations. He generalized variational principles for Korteweg-de Vries (KdV) equation and discussed non-linear Schrodinger's equation using the semi-inverse method. He described the use of VIM in developing new kind of analytical technique for solving nonlinear problems.

2. IMPLEMENTATION OF VIM

This method directly attacks the nonlinear partial differential equation without a need to find certain polynomials for nonlinear terms and gives result in an infinite series. It rapidly converges to analytical solution. This method also requires no linearization, discretization, little perturbations or restrictive assumptions. It lessens mathematical computations significantly. Consider the nonlinear partial differential equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + au - bu^q, \tag{2}$$

A correction functional is constructed as follows,

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^t \lambda(\xi)\left[\frac{\partial u_n(x,\xi)}{\partial \xi} - \frac{\partial^2 \tilde{u}_n(x,\xi)}{\partial x^2} - au_n(x,\xi) + b\tilde{u}_n^3(x,\xi)\right] \tag{3}$$

where λ is Lagrange's multiplier. Optimally, λ may be found, with the help of theory of variations. The function $\tilde{u}_n$ is restricted variation that means $\delta\tilde{u}_n = 0$.

Taking variations on both sides of Eq. (2), we get

$$\delta u_{n+1}(x,t) = \delta u_n(x,t) + \delta\int_0^t \lambda(\xi)\left[\frac{\partial u_n(x,\xi)}{\partial \xi} - \frac{\partial^2 \tilde{u}_n(x,\xi)}{\partial x^2} - au_n(x,\xi) + b\tilde{u}_n^3(x,\xi)\right]d\xi \tag{4}$$

This gives the stationary conditions

$$\begin{cases} -\lambda'(\xi) - a\lambda(\xi)|_{\xi=t} = 0, \\ \quad 1 + \lambda(\xi)|_{\xi=t} = 0. \end{cases} \tag{5}$$

The Lagrange multiplier is found as, $\lambda = -e^{-a(t-\xi)}$. (6)

Consecutive approximations u_{n+1}, n ≥ 0, are promptly found out using a discriminating function u_0. u_0 is the zeroth approximation which can be any selective function or any condition that may be used initially.

Finally, the result is achieved as $u(x,t) = \lim_{n\to\infty} u_n(x,t)$, which will be the solution of Eq. (2).

3. NUMERICAL APPROACH

(1). We first consider the non - linear convection diffusion problem

$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial x} + uu_x - u^2 + u$, subject to the initial condition: u (x,0) = e^x.

Correctional functional is,

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^t \lambda(\xi)\left[\frac{\partial u_n(x,\xi)}{\partial \xi} - \frac{\partial^2 u_n(x,\xi)}{\partial x^2} - \frac{\partial u_n(x,\xi)}{\partial x} + u_n u_{nx}(x,\xi) - u_n^2(x,\xi) + u_n(x,\xi)\right]d\xi$$

The stationary conditions follow as

$[1+\lambda] = 0$ & $[\lambda']_{\xi=t} = 0$ we get, $\lambda = -1$

By putting λ = -1, we get,

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^t (-1)\left[\frac{\partial u_n(x,\xi)}{\partial \xi} - \frac{\partial^2 u_n(x,\xi)}{\partial x^2} - \frac{\partial u_n(x,\xi)}{\partial x} + u_n u_{nx}(x,\xi) - u_n^2(x,\xi) + u_n(x,\xi)\right] d\xi$$

By using the iterative formula, we can select $u_0 = e^x$ then we have,

$$u_1 = e^x + te^x$$

$$u_2 = e^x + te^x + \frac{t^2}{2!}e^x$$

$$u_3 = e^x + te^x + \frac{t^2}{2!}e^x + .\frac{t^3}{3!}e^x + ...$$

:

The exact solution is $u(x,t) = e^{x+t}$ (fig. 1)

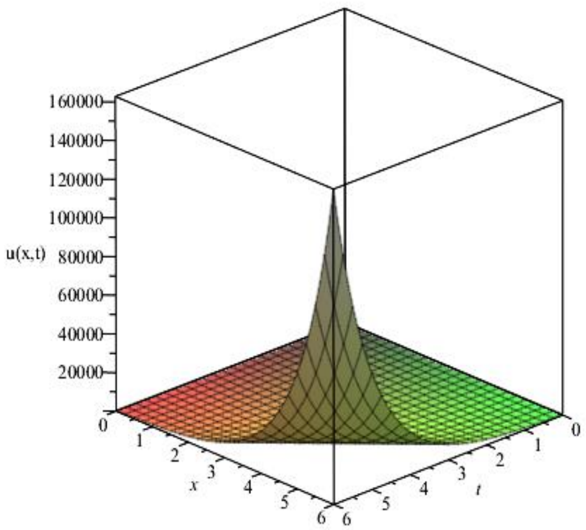


Fig. 1

(2). Next, we consider, $$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - \frac{u}{4}$$

with initial condition $$u(x,0) = \frac{x}{2} + e^{\frac{-x}{2}}.$$

Solving the stationary conditions, we get λ = -1and substituting it into iterative formula

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^t (-1)\left[\frac{\partial u_n(x,\xi)}{\partial \xi} - \frac{\partial^2 u_n(x,\xi)}{\partial x^2} + u_n(x,\xi)\right] d\xi$$

$$u_0 = \frac{x}{2} + e^{\frac{-x}{2}}$$

$$u_1 = \frac{x}{2} + e^{\frac{-x}{2}} - \frac{xt}{8}$$

$$u_2 = \frac{x}{2} + e^{\frac{-x}{2}} - \frac{xt}{8} + \frac{xt^2}{64} - .....$$

:

The result u(x,t) = $e^{-x/2} + \frac{x}{2}e^{-t/4}$ is found to be exact.(Fig. 2)

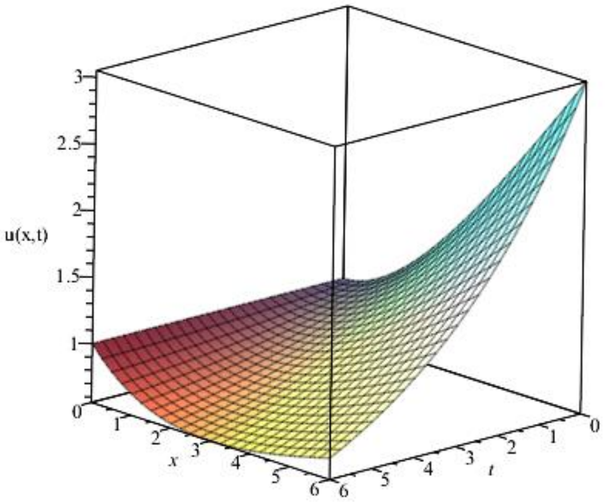


Fig. 2

(3) Consider another problem: $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - u$

with initial conditions: u(x,0) = x + e^{-x}

As before, we reach,

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^t (-1)\left[\frac{\partial u_n(x,\xi)}{\partial \xi} - \frac{\partial^2 u_n(x,\xi)}{\partial x^2} + u_n(x,\xi)\right] d\xi$$

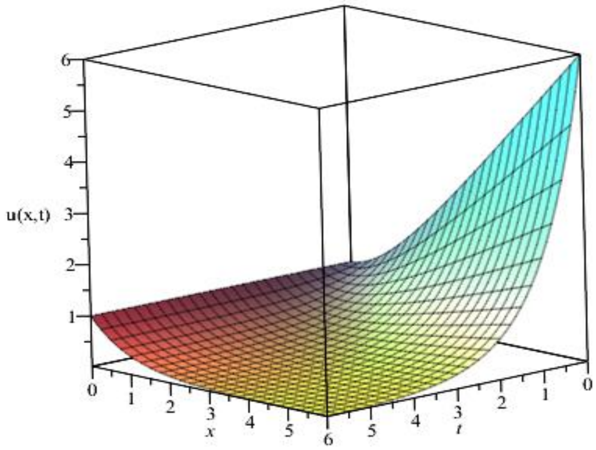
We start with $u_0 = x + e^{-x}$

$$u_1 = x + e^{-x} - xt$$

$$u_2 = x + e^{-x} - xt + \frac{xt^2}{2!} - ....$$

:

The solution is found to be u(x, t) = $e^{-x} + x\,e^{-t}$ which matches with the exact solution. (Fig. 3)



(Fig. 3)

(4) Consider the Kolmogorov – Petrovsly - Piskunov (KPP) equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - 16tu$$

Initial conditions: u(x,0) = e^{-x-4}

By iterative formula, $u_{n+1}(x,t) = u_n(x,t) + \int_0^t (-1)\left[\frac{\partial u_n(x,\xi)}{\partial \xi} - \frac{\partial^2 u_n(x,\xi)}{\partial x^2} + 16\xi u_n(x,\xi)\right]d\xi$

Starting with, $u_0 = e^{-x-4}$

$u_1 = e^{-x-4} - e^{-x-4}(8t^2 - t)$ $\quad$ $u_2 = e^{-x-4} - e^{-x-4}(8t^2 - t) + \frac{1}{2}e^{-x-4}(8t^2 - t)^2 + \ldots\ldots\ldots$

:

The result is obtained as u(x,t) = $e^{-x-4-8t^2+t}$. (Fig. 4)

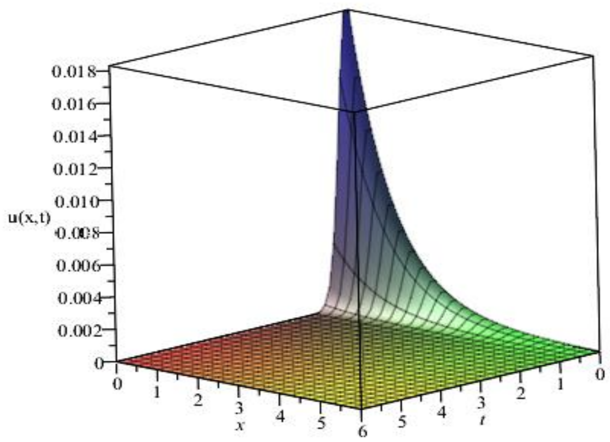


Fig. 4

(5) Similarly, We take, $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + (-1 + \cos x - \sin^2 x)u$

with initial condition u (x,0) = (1/10) e $^{\cos x - 11}$

By putting λ = -1, we get

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^t (-1)\left[\frac{\partial u_n(x,\xi)}{\partial \xi} - \frac{\partial^2 u_n(x,\xi)}{\partial x^2} - (-1 + \cos x - \sin^2 x)u_n(x,\xi)\right]d\xi$$

Using $u_0 = \frac{1}{10}e^{\cos x - 11}$ we get,

$$u_1 = \frac{1}{10}e^{\cos x - 11} - \frac{t}{10}e^{\cos x - 11}$$

$$u_2 = \frac{1}{10}e^{\cos x - 11} - \frac{t}{10}e^{\cos x - 11} + \frac{t^2}{20}e^{\cos x - 11} - ....$$

The solution is again found to be similar as exact solution (Fig. 5): u(x, t) = $\frac{1}{10}e^{\cos x - 11 - t}$.

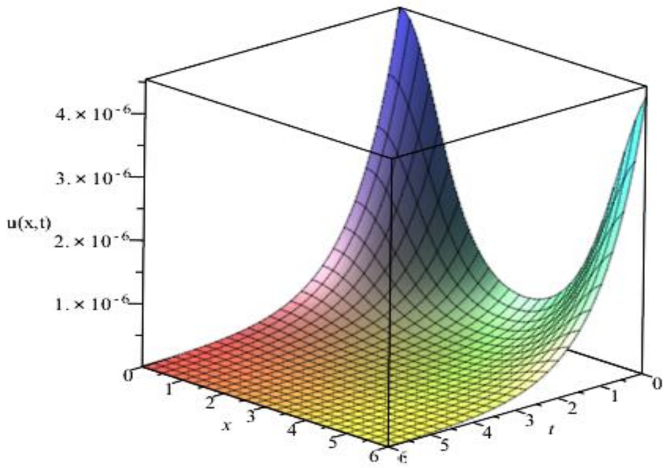


Fig. 5

4. CONCLUSION

Application of VIM is shown in solving linear and nonlinear convection-diffusion problems. This method is found to be accurate and effective for finding the exact and analytical solution of non-linear problems without the need of obtaining Adomian polynomials. It avoids the discretization error and the necessity of adding a small parameter is also not required as in HPM. This method does not need more computational work as compared to other existing methods.

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